

Parametric Oscillatory Instability in a Fabry-Perot Cavity of the Einstein Telescope with different mirror's materials

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We discuss the parametric oscillatory instability in a Fabry-Perot cavity of the Einstein Telescope. Unstable combinations of elastic and optical modes for two possible configurations of gravitational wave third-generation detector are deduced. The results are compared with the results for gravitational wave interferometers LIGO and LIGO Voyager.

INTRODUCTION

At present, there are some gravitational wave detectors in the world like LIGO, VIRGO, GEO and others. At the same time, third-generation gravitational wave detectors are under construction which have their advantages and disadvantages. Such detectors like Einstein Telescope(ET) are developed to be able to solve more ambitious goals compared to the so-called second-generation gravitational wave detectors. ET will have a sensitivity of the order of magnitude more than the sensitivity of gravitational wave detectors of the second generation. The details of this project are still under development, but the first data have already been marked in [1–3]. In this Letter, we consider two possible configurations of the ET with the Fabry-Perot mirrors made of either silicon and sapphire correspondingly.

It should be noted that the power $W = 3MW$ which circulates in the Fabry-Perot cavities of the ET will help to increase the sensitivity of the detector. In turn, some main parameters of ET are different in comparison with LIGO such as: distance L between the Fabry-Perot cavity mirrors is 10 km (compared to 4 km in LIGO); beam radius at the surfaces of the mirrors w is 12 cm (compared to 6 cm in LIGO); the mirror radius is $R = 30cm$ (2.5 larger than the laser beam radius on the mirrors). It makes optical pump mode losses due to the diffraction to be very small. The ET is planned to operate at low temperatures, which will reduce the influence of mechanical noises of various kinds (thermal noise in different parts of detectors, etc.) on the gravitational wave registration. It also planned to place the third-generation gravitational wave detector in a horizontal tunnel with two 10km arms about 800 m underground to reduce the level of the seismic sensitivity(see all other parameters of ET in Table I). For simplicity, we use the Gaussian main pump mode profile.

In turn, it is well-known that large values of circulating power W with low mechanical losses in mirrors may lead to the effect of parametric oscillatory instability(PI), which was predicted in 2001 by Braginsky et

al.[4]. This effect produces the excitation both of optical Stokes high mode with frequency ω_1 and elastic mirror mode with frequency ω_m . The energy is taken from the main optical mode with frequency ω_0 , and the sensitivity of the detector decreases. PI effect is similar to the well-known effect of light Mandelstam-Brillouin scattering. It is worth noting that the instability takes place when the condition $\omega_0 \simeq \omega_1 + \omega_m$ is fulfilled. The effect of PI in the Fabry-Perot cavities in gravitational wave antennae like ET starts due to the high optical power of up to $W \simeq 3MW$ circulating inside the arm cavities.

The presence of anti-Stokes optical modes reduces the effect of PI but can not completely suppress it[5, 6]. Parametric oscillatory is a serious problem for gravitational wave detectors, and thus you must have information about the combinations of the optical Stokes and elastic modes that may be candidates for PI[7–14]. The first observation of PI on the Advanced LIGO detectors has been made in 2015[15], thereby fully confirming the prediction of Braginsky et al.[4].

It is also well-known that the Stokes modes can be easily calculated analytically for Gaussian optical modes. On the other hand, elastic modes in the end test masses of the cylindrical mirrors can be calculated using various numerical packages(further in the calculations we use the COMSOL®). In our analysis we propose that elastic modes of only end mirror are taken into account in Fabry-Perot cavity.

THEORY OF THE PI

The condition for parametric gain \mathcal{R} when PI takes place in the Fabry-Perot cavity in the interferometer(excluding the impact of anti-Stokes mode) has the following form[4]:

$$\mathcal{R} = \frac{\Lambda_1 W \omega_1}{c L m \omega_m \gamma_m \gamma_1} \times \frac{1}{1 + \frac{\Delta^2}{\gamma_1^2}} > 1, \quad (1)$$

$$\Lambda_1 = \frac{V(\int A_0(r_\perp) A_1(r_\perp) u_z dr_\perp)^2}{\int |A_0|^2 dr_\perp \int |A_1|^2 dr_\perp \int |\vec{u}|^2 dV}, \quad (2)$$

where c is the speed of light, L is the length of the Fabry-Perot cavity, m is the mirror's mass, W is the power inside the cavity, γ_m and γ_1 are the relaxation rates of

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Parameters	Silicon	Sapphire
Pump beam profile	Gaussian	Gaussian
λ , nm	1550	1064
L ,m	10000	10000
R_1 ,m	5070	5070
R_2 ,m	5070	5070
w ,mm	120	120
W ,MW	3	3
m ,kg	230	410
T ,K	123	123
R ,cm	30	30
E ,Pa	1.64×10^{11}	4×10^{11}
ρ ,kg/m ³	2300	4000
σ	0.266	0.22

TABLE I: Parameters of the ET's Fabry-Perot cavity.

$\omega_m/2\pi$, Hz	m	Stokes mode	\mathcal{R}
17341	2	LG_{02}	3
32103	0	LG_{10}	2.2
19579	0	LG_{20}	11.3
34508	2	LG_{12}	9.2
34626	0	LG_{20}	11.1
37855	1	LG_{31}	1.2
38751	0	LG_{40}	1.4
39309	0	LG_{40}	1.7

TABLE II: The unstable elastic and Stokes optical modes in the ET with sapphire mirrors. The elastic mode is characterized by the azimuthal dependence $e^{im\varphi}$. Stokes optical modes have a Laguerre-Gauss LG_{nm} profile with the radial index n and the azimuthal index m .

the elastic and Stokes modes respectively and $\Delta = \omega_0 - \omega_1 - \omega_m$ is the detuning value. The parameter Λ_1 is the overlapping factor between the elastic and the main and Stokes optical mode, A_0 and A_1 are the optical field distributions at the surface of the mirror for the optical main mode and the Stokes mode. The vector \vec{u} is the spatial displacement of the elastic mode and u_z is the z-component of \vec{u} along the cylindrical axis. $\int dr_\perp$ are the integration over reflecting surface of the end mirror and $\int dV$ – over the mirror's volume V .

To predict the effect of PI it is necessary to know the magnitudes of the detuning and the overlapping factor(see formulas (1) and (2)). As already mentioned, the frequencies and the optical field distributions of the Gaussian optical modes is easy to calculate analytically, even in those cases when the mirrors of Fabry-Perot cavities differ slightly from each other. In [16–18] the methods of accurate calculation of frequencies and the displacement vectors for each elastic mode are discussed.

$\omega_m/2\pi$, Hz	m	Stokes mode	\mathcal{R}
32207	2	LG_{02}	1.4
19456	2	LG_{12}	1.5
34391	2	LG_{12}	2
20858	1	LG_{21}	2.7
36749	0	LG_{30}	2.9
38017	3	LG_{23}	1.7
38070	3	LG_{23}	1.8
39914	0	LG_{40}	1.5
37325	1	LG_{31}	2.3
35725	0	LG_{30}	1.2
35922	1	LG_{21}	1.3
35428	3	LG_{13}	1.4

TABLE III: The unstable elastic and Stokes optical modes in the ET with silicon mirrors. The elastic mode is characterized by the azimuthal dependence $e^{im\varphi}$. Stokes optical modes have a Laguerre-Gauss LG_{nm} profile with the radial index n and the azimuthal index m .

PI IN ET

For a Fabry-Perot cavity of ET we use the parameters shown in the Table I, where λ is the wavelength of the main optical mode, R_1, R_2 are the radii of curvature of the input and end test masses of the Fabry-Perot cavity, E – Young's modulus, ρ – density, σ – Poisson's ratio of the mirror's material. The relaxation rates of the elastic modes are calculated using loss angle $\phi = 10^{-8}$. We assume that the relaxation rates of the Stokes optical modes depend both on the coefficient of energy transmission for the input test mass and diffractive losses of the mirrors of Fabry-Perot cavities.

We have estimated the number of unstable combinations of elastic and Stokes optical modes both for the case of silicon cavity mirrors and the mirrors made of sapphire. Elastic modes frequencies and field of the deformations were evaluated by using the numerical package COMSOL® in the frequency range up to 40kHz(the number of mesh elements was around 40000). For the configuration of the ET we used the Fabry-Perot cavities with a radius of the light beam waist $w_0 \simeq 1.42$ cm. It is worth noting that all combinations of elastic and Stokes modes was deduced up to the optical modes of the ninth order taking into account the diffraction losses of the Stokes modes.

In the Tables II and III all unstable combinations of the elastic and Stokes modes in the frequency range of the elastic modes and parametric gain values for mirrors made of sapphire and silicon were performed, respectively. The number of unstable modes in the ET exceeds the number of unstable modes for a LIGO interferometer (the mass of the mirrors in the interferometer LIGO is 40 kg, the mirrors were made of fused silica). This statement can be explained by the fact that the density

of elastic modes in the spectrum is proportional to the third degree of the ratio of mirror size and the speed of sound in the material. This also leads to the fact that the number of unstable modes in the case of the mirrors made of silicon is more than for the mirrors made of sapphire (see Tables II and III). Another important reason for large number of unstable modes is the increase in the optical mode density in the spectrum for the ET due to a 2.5-fold increase in the length of its FabryPerot cavities (arm cavity length 10000m in ET vs. 4000m in Advanced LIGO).

RESULTS AND CONCLUSION

In this Letter we have analyzed two possible configurations of gravitational wave detectors of third generation like ET with the mirrors of Fabry-Perot cavities made both of silicon and sapphire and determined the number of unstable modes. At present, existing gravitational wave detectors operate at room temperatures mainly with mirrors fabricated from fused silica which has a high value of mechanical Q-factor ($Q \simeq 2 \times 10^7$). However, in order to increase the sensitivity of the third-generation detectors larger values of Q-factors are required, and the detectors must operate at the temperatures substantially lower than room temperature. Unfortunately, the quality factor Q of fused silica decreases with decreasing of temperature [19, 20]. At the same time, silicon has good mechanical properties at low temperatures: high value of quality factor (Q-factor reaches 10^9), a small thermal expansion coefficient, etc. These facts make silicon a suitable material for use in third-generation gravitational wave detectors like ET.

However, silicon has a high absorption coefficient at the wavelength 1064nm, which is equal to $\alpha \simeq 10\text{cm}^{-1}$ (for fused silica $\alpha \simeq 2.5 \times 10^{-7}\text{cm}^{-1}$) [21]. In [22] the authors proposed to use a silicon with a laser at a wavelength of 1550nm, because silicon has a small optical absorption $\alpha \simeq 10^{-8}\text{cm}^{-1}$ on this wavelength. Another important problem for using silicon or sapphire is their anisotropic properties.

The results of this Letter suggest that using mirrors made of silicon or sapphire will increase the number of parametric instabilities in the Einstein Telescope as compared to the LIGO interferometer. In the frequency range of elastic modes up to 40kHz we have found 8 unstable combinations for sapphire mirrors and 12 combinations for silicon mirrors correspondingly.

It is also useful to compare the number of unstable modes both in ET and in LIGO Voyager (blue design) for the mirrors made of silicon [23]. LIGO Voyager Blue design have practically the same parameters: laser wavelength is 2000nm, the mass of silicon mirrors with low acoustic loss is 204kg at the operating temperature of 123K and the arm cavity power 3MW. In [23] the para-

metric gains \mathcal{R} for different elastic modes have been calculated. At cavity power of 3MW in LIGO Voyager there are 1161 modes with $\mathcal{R} > 0$ of which two modes have $\mathcal{R} > 1$. The maximum parametric gain is 76. Blair et al. have found the number of unstable elastic modes for different radii of curvature (ROC) of input test mass of Fabry-Perot cavity. The number of unstable modes varies from 2 to 5 for different ROCs. It is easy to say that this result is in good agreement with the results calculated in this Letter. The number of unstable modes for ET with silicon test masses preliminary 2.5 times more than in LIGO Voyager because the ratio of arm cavity lengths is $L_{ET}/L_{Voyager} = 2.5$, and the optical mode density increases preliminary in the same manner.

In general, the same results were deduced in [24] for 10000m ET arm cavity length with fused silica mirrors. The number of unstable modes varied from 5 to 9 for different ROCs. We have to say that our results have only stochastic character because of variations in elastic parameters (Young's modulus, density and others). Therefore, the values of parametric gains can slightly fluctuate that influences the number of possible PI unstable modes. In this case the experimental results are required.

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- [1] Hild S, Chelkowski S and Freise A *arXiv:0810.0604*
 - [2] Yamamoto K and et al 2009 <http://www.et-gw.eu>, Document ET-029-09
 - [3] Abernathy M and et al 2011 <http://www.et-gw.eu>, Document ET-0106A-10
 - [4] Braginsky V B, Strigin S E and Vyatchanin S P 2001 *Physics Letters A* **287** 331
 - [5] D'Ambrosio E and Kells W 2002 *Physics Letters A* **299** 326
 - [6] Braginsky V B, Strigin S E and Vyatchanin S P 2002 *Physics Letters A* **305** 111
 - [7] Zhao C, Ju L, Degallaix J, Gras S and Blair D G 2005 *Phys. Rev. Lett.* **94** 121102
 - [8] Strigin S E and Vyatchanin S P 2007 *Physics Letters A* **365** 10
 - [9] Gras S, Blair D G and Zhao C 2009 *Class. Quantum Grav.* **26** 135012
 - [10] Evans M, Barsotti L and Fritschel P 2010 *Physics Letters A* **374** 665
 - [11] Ju L, Zhao C, Gras S, Degallaix J, Blair D G, Munch J and Reitze D H 2006 *Phys. Lett. A* **355** 419
 - [12] Ju L, Gras S, Zhao C, Degallaix J and Blair D G 2006 *Phys. Lett. A* **354** 360
 - [13] Gras S, Blair D G and Zhao C 2009 *Classical and Quantum Gravity* **26** 135012

- [14] Strigin S E and Vyatchanin S P 2007 *Quantum Electronics* **37** 1097
- [15] Evans M and et al 2015 *Phys. Rev. Lett.* **114** 161102
- [16] Strigin S E, Blair D G, Gras S and Vyatchanin S P 2008 *Physics Letters A* **372** 5727
- [17] Strigin S E 2008 *Physics Letters A* **372** 6305
- [18] Meleshko V V, Strigin S E and Yakymenko M S 2009 *Physics Letters A* **373** 3701
- [19] Travasso F and et al 2007 *Europhys. Lett.* **80** 50008
- [20] Vacher R and et al 2015 *Phys. Rev.* **72** 214205
- [21] Green M and Keevers J 1995 *Progr. Photovoltaic Res.* **3** 189
- [22] Schnabel R and et al *arXiv:0912.3164*
- [23] Zhang J, Zhao C, Ju L and Blair D *arXiv:1609.01505*
- [24] Strigin S E 2015 *Physics Letters A* **379** 1671